erature drops along the length are not less than 30°K for the same flow rate of cooling liquid.

On the basis of the above we can recommend this method for obtaining thermal conditions for certain elements in the instrumentaion field.

NOTATION

x, coordinate, m; T, T^*, T_{in}, temperatures of the element surface (actual and assigned) and of air at the channel inlet, °K; $\alpha(x)$, coefficient of convective heat transfer, $W/m^2 \cdot K$; $\lambda(x)$, thermal conductivity of the element material, W/m·K; S(x) and Π, area of cross section and perimeter of the element being cooled, m^2, m; q(x), specific power of the heat sources in the element, W/m; W_i, electrical power supplied to section i, W; O_i, loss flux, W; F, section area, m^2; F_c, area of the internal cone surface, m^2; ε and ε_c, emissivities of the rod and cone surfaces; ε_r, reduced emissivity; D(x), diameter of the internal channel wall, m; d, diameter of the element being cooled, m; λ, thermal conductivity of the liquid, W/m·K; ν, viscosity of the liquid, m^2/sec; M(x), flow area of the channel, m^2; T_i, T_w, temperatures of section i of rod 1 and of the wall of channel 2 (Fig. 1), respectively, °K.

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HEAT TRANSFER IN TURBULENT FREE CONVECTION AROUND A HORIZONTAL NONISOTHERMAL CYLINDER

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UDC 536.25

The heat transfer of a horizontal cylinder in turbulent free convection and for a quadratic law of temperature variation on its surface is investigated numerically: $10^9 \leq Ra \leq 10^{13}$, Pr = 0.71.

Heat transfer with free convection at horizontal circular cylinders has been studied repeatedly both experimentally and theoretically [1-5]. Development of a turbulent flow regime is considered below for nonisothermal boundary conditions that can hold near the surface of powerful thermal power plant elements [6].

The mathematical model of the process is the Reynolds equation in the Boussinesq approximation which reduces to a system of three differential equations of identical structure [5] after going over to dimensionless form and using the new independent variable ξ = ln R.

$$\exp(-2\xi)\left\{\frac{\partial}{\partial\varphi}\left(\gamma\Phi\frac{\partial\psi}{\partial\xi}-\zeta\frac{\partial(\eta\Phi)}{\partial\varphi}\right)-\frac{\partial}{\partial\xi}\left(\gamma\Phi\frac{\partial\psi}{\partial\varphi}\right)+\zeta\frac{\partial(\eta\Phi)}{\partial\xi}\right)\right\}=S, \quad (1)$$

where the average axial component of vorticity, the stream function, and temperature are considered, respectively, as the desired function Φ. The specific form of the coefficients of (1) is represented in Table 1. The following scales, r_0, a/r_0, $t_m - t_f$, are chosen for the coordinate, the average velocity vector component, and the temperature.

The problem was solved for the boundary conditions

$$\xi=0 \quad \partial\psi/\partial\xi=\psi=0,\ T=T(\varphi); \qquad \xi=5 \quad T=\Omega=\psi=0; \qquad \varphi=0,\pi \quad \psi=\Omega=\partial T/\partial\varphi=0.$$

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TABLE 1. Coefficients of Equation (1)

Φ	γ	ξ	η	S
Ω	Pr^{-1}	1	$1+\frac{\varepsilon}{\nu}$	$\exp(-\xi)\frac{\mathrm{Ra}}{8}\left(\frac{\partial T}{\partial \xi}\sin\varphi+\frac{\partial T}{\partial \varphi}\cos\varphi\right)+S_\omega$
ψ	0	1	1	Ω
T	1	$1+\frac{\varepsilon}{\nu}\frac{\mathrm{Pr}}{\mathrm{Pr}_t}$	1	0

The computation domain was a half-ring with inner radius r_0 and outer radius $r_0\exp(5) = 148.41r_0$. Validity of the assumption of negligibly small values of the dependent variables on so large a radius is given a foundation in [4].

A quadratic dimensionless temperature distribution over the outline of the cylinder transverse section was studied

$$T(\varphi) = \frac{\sigma - 1}{\pi^2}\varphi^2 + j, \tag{2}$$

where

$$\sigma = T_u - T_\ell, \; f = \begin{cases} 1, & 0 \leqslant \sigma \leqslant 1, \\ 2-\sigma, & 1 \leqslant \sigma \leqslant 2, \end{cases}$$

The magnitude of the constant σ in (2) varied between 0 and 2, which permitted enclosing the whole range of temperature differences between the upper T_u and lower T_l points of the outline (one of which is assumed unity) including for $\sigma = 1$, the case of isothermal surface heating.

An explicit finite-difference method whose details are contained in [4, 5] was utilized in solving the problem.

Two modeling sections can provisionally be extracted: the intrinsic boundary layer on the surface and the vertical freely rising jet above the cylinder.

The turbulent viscosity in the near-wall domain was determined by the model of the mixing path length. The action of just one of the four substantial fluctuating stress components was here taken into account in the plane case. However, the errors induced by such an assumption are small because of the smallness of the corresponding terms in the system (1) within the boundary-layer limits. More complex models with additional equations for the turbulence parameters are free of such a disadvantage. Meanwhile, the experience of previous investigations [7-9] shows that even so simple, and therefore, an economic a model assures good agreement with experimental results in the case of simple flows.

Different modifications of the mixing path length model utilized earlier to solve the free convection problem were compared in [9] in a study of the structure of a turbulent jet rising along an isothermal wall. The analysis performed showed that the greatest accuracy held for a computation using the relationships recommended by Cebeci and Khattabom [8], which can be represented as follows when taking account of the notation used

$$\frac{\varepsilon}{\nu} = \frac{L^2}{\mathrm{Pr}}\left|\frac{\partial^2\psi}{\partial\xi^2}\right|, \tag{3}$$

where

$$L = \begin{cases} L_i = 0.4(\exp(\xi)-1)\left(1-\exp\left((1-\exp(\xi))\sqrt{\frac{1}{\mathrm{Pr}}\frac{\partial^2\psi}{\partial\xi^2}}\Big/26\right)\right), \\ L_0 = 0.075\,\delta/r_0, \; L_0 < L_i. \end{cases}$$

The distance along the normal to the cylinder surface to the point where the maximum holds for the distribution of the circumferential velocity vector component was taken as the boundary layer thickness δ.

This model was modified to take account of the influence of effects associated with the surface curvature: an additional correction dependent on the Richardson number was introduced [10-12].

$$L_* = L(1 - c\,\mathrm{Ri}),$$

$$\frac{L_*}{L} = \begin{cases} 1 - 2c\,\dfrac{\partial\psi}{\partial\xi} \Big/ \dfrac{\partial^2\psi}{\partial\xi^2}, & \dfrac{\partial^2\psi}{\partial\xi^2} > 0.3\,\dfrac{V_{\varphi m}}{\Delta}, \\ 1 - 2c\,\dfrac{\partial\psi}{\partial\xi} \Big/ \left(0.3\,\dfrac{V_{\varphi m}}{\Delta}\right), & \Delta = \delta/r_0. \end{cases} \tag{4}$$

Selection of the constant c in (4) is not unique: its magnitude depends on the kind of flow and fluctuates between 1.5 and 6 for different data [10-12]. A number of experiments indicates [12] that c is close to 1.5 for problems of external flow around a cylinder. Precisely this value was indeed utilized in the computations. Let us note that as trial calculations showed, variation of the magnitude of the constant c in (4) does not result in noticeable changes in the local heat transfer characteristics.

A freely rising jet above a cylinder apparently has slight influence on the flow characteristics near the surface. The accuracy of computing the hydrodynamic variables is not high since the mesh spacing in a radial direction increases and becomes nonuniform with distance from the surface. For this reason, the Prandtl constant viscosity formula was used to model the turbulent transport [13]:

$$\frac{\varepsilon}{\nu} = 0{,}0246\,BV_{rm}/\mathrm{Pr}.$$

The magnitude of the turbulent Prandtl number does not change in each of the domains and was assumed equal to 0.9 in the boundary layer and 0.5 in the freely rising jet.

The computations were performed on nonuniform 41 × 41 difference mesh with node compaction in the domains where the solution has large gradients. Thus, the relative mesh spacing in the near-wall domain was constant ($h_\xi = 0.001$) and the velocity went from four to eight nodes in the radial direction on the section to the maximum point.

The computed fields of the isostreamlines and isotherms in the domain of high Rayleigh numers were qualitatively similar to those obtained earlier for the laminar flow mode [6].

Distributions of the dimensionless local heat fluxes along the outline of an isothermal cylinder transverse section are represented in Fig. 1. It is seen that the nature of the heat transfer of the cylinder surface in the case of a turbulent free convection mode has a number of substantial features. Firstly, it is the abrupt growth in the intensity and displacement (by more than 100-120°) under the effect of turbulent transport of the maximal heat transfer zone. The quantity $\mathrm{Nu}/\mathrm{Ra}^{1/3}$ changes as the Rayleigh number increases. Indeed, a low-intensity freely convective flow exists near the lower point of the transverse section outline for arbitrarily large values of Ra, which also specifies small values of ε and a reduction in the relative local heat transfer as Ra grows.

The results obtained are in satisfactory agreement with the computations of Farouk and Gucceri [14] (Fig. 1), executed in the range $10^8 \leqslant \mathrm{Ra} \leqslant 10^{10}$ by using a more complex k-ε turbulence model: the maximal difference is about 11%. The smaller magnitudes of the local thermal fluxes in [14] are apparently explained by the fact that the k-ε model takes account of the presence of the laminar section for small angles φ. Let us note that computations by both models yield practically identical values of $\overline{\mathrm{Nu}}$.

As is seen from Fig. 1, the maximal values of Nu shift toward smaller angles as Ra grows. The same tendency is also noted in [14]. The data of the calculations are described by the following regression equation constructed by the method of correlation analysis, where the maximal deviation of the individual computed points from the average curve does not exceed 3%:

$$\frac{\mathrm{Nu}}{\mathrm{Ra}^{1/3}} = 0{,}383 - 0{,}067\,\lg\mathrm{Ra}\,(1 - 0{,}0421\,\lg\mathrm{Ra} + 0{,}13\cos\varphi - 0{,}184\sin\varphi) + $$
$$+ 0{,}108\cos\varphi\,(1 + 0{,}47\cos\varphi - 0{,}626\sin\varphi).$$

Results on the mean heat transfer of the cylinder surface were processed analogously. The following dependence

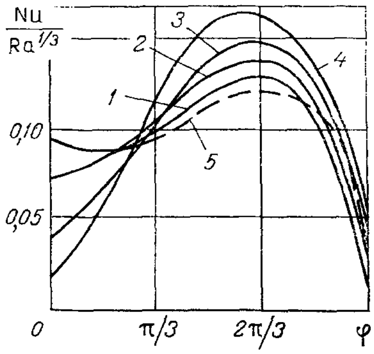

Fig. 1

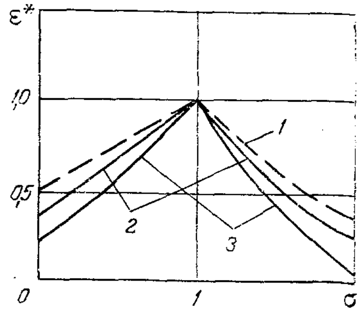

Fig. 2

Fig. 1. Local heat transfer of an isothermal cylinder: 1) Ra = 10^9; 2) Ra = 10^{11}; 3) Ra = 10^{12}; 4) Ra = 10^{13}; 5) Ra = 10^9 numerical solution [14].

Fig. 2. Dependence of the function ε^* on the change in temperature head as the Rayleigh number increases: 1) laminar mode [5]; turbulent mode; 2) Ra = 10^9; 3) 10^{13}.

TABLE 2. Comparison of Data on the Mean Nusselt Number Nu for an Isothermal Cylinder

References	lg Ra				
	9	10	11	12	13
Bosworth [15]	136,8	284,7	599	1269	2623
Mikheev [16]	135	290,8	626,6	1350	2908
Churchill and Chu [17]	116	240	505	1069	2276
Wrightby and Hollands [18]	107	221	466	992	2123
Kuehn and Goldstein [3]	101	216	465	1001	2155
Present paper	103	226,2	496	1083	2511

TABLE 3. Polynomial Coefficients

Range of variation of σ	φ	A_0	A_1	A_2	A_3	A_4	A_5
$0 \leqslant \sigma < 1$	0	0,0310	—0,0291	0,0820	—0,1071	0,0479	0,0
	0,5235	0,0550	—0,0049	0,0609	—0.1192	0,0734	0,0
	1,047	0,0721	0,1059	—0,2243	0,3953	—0,2245	0,0
	1,57	0,0386	0,0822	0,2629	—0,3705	0,1472	0,0
	2,094	—0,00052	—0.1101	1,9120	—4,3866	4,0635	—1,3111
	2,618	—0,0060	—0,2536	1,56875	—0,5187	—3,3545	2,8580
	3,142	—0,0075	—0,0105	0,11325	—0.03598	—0,2669	0,2468
$1 < \sigma \leqslant 2$	0	0,2586	—0,6824	0,7494	—0,3641	0,0638	0,0
	0,5235	—0,4820	1,2664	—0,9236	0,20457	0,0	0,0
	1,047	0,4664	—0,6895	0,6270	—0,3071	0,0247	0,01429
	1,57	—0,5144	1,7712	—1,4321	0.3367	0,0	0,0
	2,094	0,4735	—0,3753	0,0693	0,0	0,0	0,0
	2,618	—2,4029	7,7846	—8,6526	4.1164	—0,7135	0,0
	3,142	—0.2013	0,6648	—0,6467	0,2589	—0,0371	0,0

$$\overline{\mathrm{Nu}} = (0{,}092 + 0{,}0016\ \lg \mathrm{Ra})\, \mathrm{Ra}^{1/3}$$

for $10^9 \leqslant \mathrm{Ra} \leqslant 10^{13}$ was obtained with 2% error.

A comparison between the computed mean Nusselt numbers and those calculated by the criterial formulas proposed earlier is presented in Table 2. The numerical results are in good agreement with empirical dependence in the whole investigated range of Rayleigh number variation.

Intensification of the heat transport in the turbulent flow mode exerts substantial influence on the regularities of the cylinder heat transfer even under nonisothermal boundary conditions. The degree of this influence is magnified as the Rayleigh number grows. An increase in the mean temperature of the fluid running over the upper part of the cylinder surfact results in an increase in the extent of the section with opposite thermal flux vector

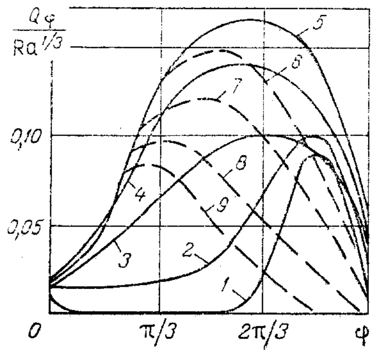

Fig. 3. Local heat transfer of a nonisothermal cylinder for large Rayleigh numbers: 1) $\sigma = 1.8$; 2) 1.6; 3) 1.4; 4) 1.5; 5) 1; 6) 0.8; 7) 0.6; 8) 0.4; 9) 0.2.

direction for a negative temperature drop along the transverse section outline, and in a reduction in the local temperature heat in the case of positive heat transfer ($\sigma > 1$). Consequently, a stronger diminution in the cylinder mean heat transfer (Fig. 2) is observed, as compared with the laminar flow mode.

Computed values of the coefficient of diminution ε^* in the Rayleigh number range investigated are approximated by the following similarity equations:

$$\varepsilon^* = 0.88 + 0.132\sigma^2 + 0.0472(\sigma - 1)\lg \mathrm{Ra}, \quad 1 \geqslant \sigma \geqslant 0;$$

$$\varepsilon^* = 2.28 + 0.00184(\lg \mathrm{Ra})^2 + 1.38\sigma(0.24\sigma - 0.03\lg \mathrm{Ra} - 1), \quad 2 \geqslant \sigma \geqslant 1.$$

Here the maximum error is 2.5%.

Because the turbulence model used did not permit computation of the flow transition zone, a statistical analysis of the obtained results on the local heat transfer of a nonisothermal cylinder was performed only in the domain of high Rayleigh numbers Ra = 10^{12}–10^{13}. Dependences of the form

$$Q_\varphi \mathrm{Ra}^{-1/3} = \sum_{i=0}^{4} A_{\varphi i} \sigma^i$$

have been established for a number of values of the polar angle, where $\varphi = \pi j/6$, $j = 0, 1, 2, \ldots, 6$; $0.2 \leqslant \sigma \leqslant 1.8$. The coefficients $A_{\varphi i}$ are presented in Table 3. The last formulas permit determination of the magnitude of the dimensionless thermal fluxes at individual points of the outline, and at any intermediate point by using interpolation.

The computed distributions Q_φ along the cylinder transverse section outline are shown in Fig. 3 for different values of the parameter σ for Ra = 10^{13}. In addition, a substantially more narrow range of variation of the independent variables $10^{11} \leqslant \mathrm{Ra} \leqslant 10^{13}$; $0.6 \leqslant \sigma \leqslant 1.4$ holds for the formulation of heat transfer boundary conditions to the almost cylindrical housing surfaces of steam turbines. The distribution of the local heat transfer coefficients can be determined in this case by means of the following approximate formulas (10% error)

$$\sigma \geqslant 1 \quad Q_\varphi/Q_1 = 1.32 + 0.11\cos\varphi(\cos\varphi - 3.4\sigma) + 0.0037\lg \mathrm{Ra}(\lg \mathrm{Ra} + 10.7\cos\varphi - 22\sigma),$$

$$\sigma \leqslant 1 \quad Q_\varphi/Q_1 = 1.07 - 0.11\cos\varphi(\cos\varphi + 7.5\sigma - 7.7) - 0.076\lg \mathrm{Ra}(1 - \sigma).$$

Therefore, the nonisothermy of the cylinder surface in the case of a turbulent flow mode with natural convection exerts substantial influence on the local and mean heat transfer coefficients. This influence can be taken into account on the basis of the proposed dependences. Further refinements can be obtained on the basis of more perfect turbulence models.

NOTATION

φ, polar angle measured from the lower point of the cylinder transverse section outline, rad; r, radial coordinate; r_0, d, cylinder radius and diameter; $R = r/r_0$, $T = (t - t_f)/(t_m - t_f)$, dimensionless average temperature; V_r, V_φ, dimensionless radial and tangential mean velocity vector components; Ω, ψ, dimensional axial component of the velocity vector and the stream function; $V_r = -\exp(-\xi)\partial\psi/\partial\varphi$; $V_\varphi = \exp(-\xi)\partial\psi/\partial\xi$, $\Omega = \exp(-\xi)(\exp(-\xi)\partial(\exp(\xi)V_\varphi)/\partial\xi -$

$\partial V_r/\partial\varphi$); a, ν, λ, β, respectively, thermal diffusivity, kinematic viscosity, heat conduction, and volume expansion coefficients of a fluid; g, free-fall acceleration; $Ra = g\beta(t_m - t_f)d^3/(\nu a)$, Rayleigh number; $Pr = \nu/a$, Prandtl number; ε, coefficient of turbulent viscosity; L, dimensionless mixing path length; c, empirical constant to take account of the influence of surface curvature; $Ri = 2(V\ /r)/(\partial V_\varphi/\partial\xi)$, Richardson number, Pr_t, turbulent Prandtl number; $B = b/r_0$, b, distance between the jet axis and a point at which $V_r/V_{rm} = 0.5$, $S_\omega = 2\left(\frac{\partial V_r}{R\partial\varphi} - \frac{V_\varphi}{R}\right)\frac{\partial^2\varepsilon}{\partial R^2} - 4\frac{\partial V_r}{\partial R}\frac{\partial^2\varepsilon}{R\partial\varphi\partial R} - 2\frac{\partial V_\varphi}{\partial R}\frac{\partial^2\varepsilon}{R^2\partial\varphi^2} + \frac{4}{R}\frac{\partial V_r}{\partial R}\frac{\partial\varepsilon}{R\partial\varphi} - \frac{2}{R}\times\frac{\partial V_\varphi}{\partial R}\frac{\partial\varepsilon}{\partial R}$ is the additional term in the velocity vortex transport equation; q_φ, $\bar{q}$, local and mean heat fluxes; $Q_\varphi = q_\varphi d/(\lambda(t_m - t_f))$, $\bar{Q} = \bar{q}d/(\lambda(t_m - t_f))$, local and mean dimensionless heat fluxes; $Q_1 = Nu$; $Nu = \alpha d/\lambda$, Nusselt number; α, heat-transfer coefficient; $\varepsilon^* = \bar{Q}/\bar{Q}_1$. Subscripts: m, maximum value; 1, isothermal case; f, far from the cylinder surface, and φ corresponds to a function of the polar angle.

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